DBL-003-1015003 Seat No. ________

B. Sc. (Sem. V) (CBCS) (W.E.F. 2016) Examination

June – 2022

Mathematics : MATH-07(A)

(Boolean Algebra & Complex Analysis-1)

Faculty Code : 003
Subject Code : 1015003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) Attempt any five questions out of ten questions.
(ii) Figures written on the right indicate full marks of the question.

1 (a) Attempt the following objective type questions : **4**
(1) Define : Partial order relation.
(2) For the poset (S_{30}, D), 10'=_____.
(3) If (S_6, D) is lattice then glb{2,3}=_____.
(4) The minimal element of poset ({2,3,4,5,6}, /)=_____.
(b) In a complemented distributive lattice prove that $a' \vee b = 1 \Rightarrow b' \leq a'$. **2**
(c) State and prove modular inequality. **3**
(d) Prove that direct product of two lattice is also a lattice. **5**

2 (a) Attempt the following objective type questions. **4**
(1) Define : Antisymmetric relation.
(2) Define : Meet and Join.
(3) For the poset $(P(A), \subset)$ where $A = \{a, b\}$ then find greatest element and least element.
(4) For the poset (S_{30}, D), $2' =$ ____ .
(b) If $(L, \leq)$ be a lattice then prove that for every $a, b \in L$, $a \cap (a \vee b) = a$, **2**
(c) State and prove isotonicity property. **3**
(d) Let A be a nonempty set and $P(A)$ be the power set of A. $X, Y \in P(A) = L$, $X * Y = X \cap Y$ and $X \oplus Y = X \cup Y$ then prove that $(L, *, \oplus)$ is a lattice. **5**

3 (a) Attempt the following objective type questions. 4

(1) Define : Boolean Algebra.

(2) For Boolean Algebra $a \oplus 1 =$ _____.

(3) Sum of all minterms of n variables is _____.

(4) Define : Minterm

(b) Let $(B, *, \oplus, ', 0, 1)$ be a Boolean Algebra then prove that $\forall a, b \in B \;\; a \le b \Rightarrow a * b' = 0$. 2

(c) If a and b are distinct atems of Boolean Algebra $(B, *, \oplus, ', 0, 1)$ then prove that $a * b = 0$. 3

(d) State and prove unique representation theorem of Boolean Algebra. 5

4 (a) Attempt the following objective type questions : 4

(1) Define : Atom in Boolean Algebra.

(2) If $(L, *, \oplus, 0, 1)$ is bounded lattice then $a \oplus 0 =$ _____.

(3) Find the atoms of Boolean Algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(4) Define : Qube array representation.

(b) Define : Boolean Homomorphism & Isomorphism. 2

(c) Write all the minterms of the two and three variables. 3

(d) State and prove D'Morgan's law for the Boolean Algebra $(B, *, \oplus, ', 0, 1)$. 5

5 (a) Attempt the following objective type questions : 4

(1) Define : Limit of complex variables function.

(2) Show that $f(z) = z$ is analytic.

(3) $\lim\limits_{z \to \infty} \dfrac{2z - 5}{z - 2i} =$ _____.

(4) State Laplace equation in polar form.

(b) Show that $u = \frac{1}{2} \log\left(x^2 + y^2\right)$ is harmonic function. 2

(c) Prove that an analytic function of constant modulus is also constant in its domain. 3

(d) It $f(z) = u + iv$ is an analytic function then in usual notation prove that $\nabla^2 |f(z)|^2 = 4 |f'(z)|^2$. 5

6 (a) Attempt the following objective type questions : 4

(1) Define : Analytic function.

(2) If $f(z) = u(x, y) + i\, v(x, y)$ is analytic function then $f'(z) =$ ____.

(3) Define : conjugate Harmonic function.

(4) State Cauchy-Riemann condition in polar form.

(b) Prove that $u = \log r$ is harmonic and find its conjugate. **2**

(c) If $f(z) = e^{-x}(\cos y - i \sin y)$ then prove that $f''(z) = f(z)$. **3**

(d) State and prove necessary condition for a complex function $f(z) = u(x, y) + i\, v(x, y)$ to be an analytic. **5**

7 (a) Attempt the following objective type questions : **4**

(1) Define : Closed curve.

(2) If $c : |z| = 1$ then $\int \frac{z}{2z-1} dz =$ _____.

(3) If $c : |z-1| = 2$ then $\int_c \frac{z}{z-2} dz =$ _____.

(4) If $c : |z-2| = \frac{1}{2}$ then $\int_c \frac{dz}{z-3} =$ _____.

(b) Find $\int_0^{2+i} z^2 dz$. **2**

(c) In usual notation prove that $\left| \int_c f(z)\, dz \right| \leq ML$. **3**

(d) Let C be the circle $|z| = R\,(R > 1)$ described in the counter clock direction then show that **5**

$$\left| \int_c \frac{\log z}{z^2} dz \right| < 2\pi \left(\frac{\pi + \log R}{R} \right).$$

8 (a) Attempt the following objective type questions : **4**

(1) Define : Jordan arc.

(2) If $c : z - z_0 = r_0 e^{i\theta}$ then $\int_c \frac{dz}{z - z_0} =$ _____.

(3) If L is length of contour c then $L =$ _____.

(4) If $c : |z-2| = 5$ then $\int_c \frac{dz}{z-3} =$ _____.

(b) Find $\int_c \frac{dz}{z^2+4}$, $\quad c : |z-i| = 2$. **2**

(c) Prove that $\left| \int_c \frac{1}{z^2} dz \right| \leq 4\sqrt{2}$ where c is the line segment joining $z = i$ to $z = 1$. **3**

(d) State and prove Cauchy integral formula. **5**

9 (a) Attempt the following objective type questions : **4**

(1) State Cauchy inequality.

(2) If $c:|z|=1$ then $\int_c \frac{\cosh z}{z^4}\,dz =$ ____ .

(3) If $c:|z|=1$ then $\int_c \frac{e^z}{z^3}\,dz =$ _____ .

(4) State maximum modulus theorem.

(b) Evaluate $\int_c \frac{5z^2-7z+3}{(z-1)^2}\,dz$; $c:|z|=2$. **2**

(c) Find $\int_c \frac{dz}{z^2\left(z^2+9\right)}$; $c:|z-2|=\frac{1}{2}$. **3**

(d) State fundamental theorem of Algebra. **5**

10 (a) Attempt the following objective type questions : **4**

(1) State Cauchy-integral formula for n^{th} derivative.

(2) State Liouville's theorem.

(3) If $c:|z|=1$ then $\int_c \frac{e^{2z}}{z^3}\,dz =$ _____ .

(4) If $c:|z-2|=\frac{1}{2}$ then $\int_c \frac{dz}{(z-3)^2} =$ ____ .

(b) Evaluate $\int_c \frac{e^2}{(z-4)(z-2)}\,dz$, $c:|z|=3$. **2**

(c) Evaluate $\int_c \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}\,dz$, $c:|z|=1$. **3**

(d) State and prove Morera's theorem. **5**